Analysis of a Delta Wing with Leading-Edge Flaps

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Nomenclature

C_N = normal force coefficient
D = drag
F = complex potential
k = ratio of main wing span length to total span length
L = lift
N = normal force
R = resultant force
U = freestream velocity
V = complex velocity
α = angle of attack
δ = flap deflection angle
Γ = vortex strength
ϵ = half-apex angle of a main wing
η,σ = complex variables defining physical transformed plane
ρ = density
$(\bar{\ })$ = indicates complex conjugate

Subscripts
F = flap
o = lower limit of integration
s = separation point
v = vortex position
w = main wing
y = direction normal to main wing

Abstract

THE effect of a leading-edge flap on the aerodynamic characteristics of a low-aspect-ratio delta wing has been studied using vortex-feeding-sheets singularity systems to represent the separated flow. The analysis was performed in the crossflow plane using the Schwarz-Christoffel transformation. Particular attention was paid to the influence of flap deflection on lift and drag. It was found that both lift and drag decrease during flap deflection, while the lift-to-drag ratio increases. The main reason for lift reduction is the partial suppression of the vortex system from the leading edge as a result of flap deflection, while drag reduction originates in a propulsive component of the pressure force acting on the deflected flaps.

Contents

Extending Brown and Michael's idea[1] to the flapped wing, the model replaces the vortex sheets originating at the leading edges and the flap junctions by two pairs of vortex-feeding-sheet systems, as illustrated in Fig. 1. Due to symmetry, only half of the singularity system is considered. To each vortex-feeding-sheet arrangement a global force equilibrium condition is imposed. Under the assumption of conical flow, this condition is

$$\frac{U\epsilon}{k}(2\sigma_v-\sigma_s)=V_v \tag{1}$$

where

$$\bar{V}_v=\frac{\mathrm{d}}{\mathrm{d}\sigma}\left[F-\frac{i\Gamma}{2\pi}\log(\sigma-\sigma_v)\right]_{\sigma=\sigma_v} \tag{2}$$

The complex velocity potential in the cross-flow plane is obtained by mapping the wing trace into the real axis of the transformed plane, as shown in Fig. 2. This is done using the Schwarz-Christoffel transformation. With the three Riemann mapping conditions,

$$\eta_4=0, \qquad \eta_o=\eta_2, \qquad \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\eta}\right|_{\sigma \text{ or } \eta\to\infty}=1 \tag{3}$$

the transformation function is

$$\sigma=\int_{\eta_2}^{\eta}\frac{\eta}{\sqrt{(\eta-\eta_2)(\eta-\eta_6)}}\left(\frac{\eta-\eta_5}{\eta-\eta_3}\right)^{\delta/\pi}\mathrm{d}\eta \tag{4}$$

The coefficients η_2, η_3, η_5, and η_6 are determined numerically by matching the points σ_3, σ_4, σ_5, σ_6 to the corresponding points η_3, 0, η_5, η_6.

The complex potential in the transformed plane is

$$F(\eta)=U\alpha\eta+\frac{i\Gamma_1}{2\pi}\log\left(\frac{\eta-\eta_{v1}}{\eta-\bar{\eta}_{v1}}\right)+\frac{i\Gamma_2}{2\pi}\log\left(\frac{\eta-\eta_{v2}}{\eta-\bar{\eta}_{v2}}\right) \tag{5}$$

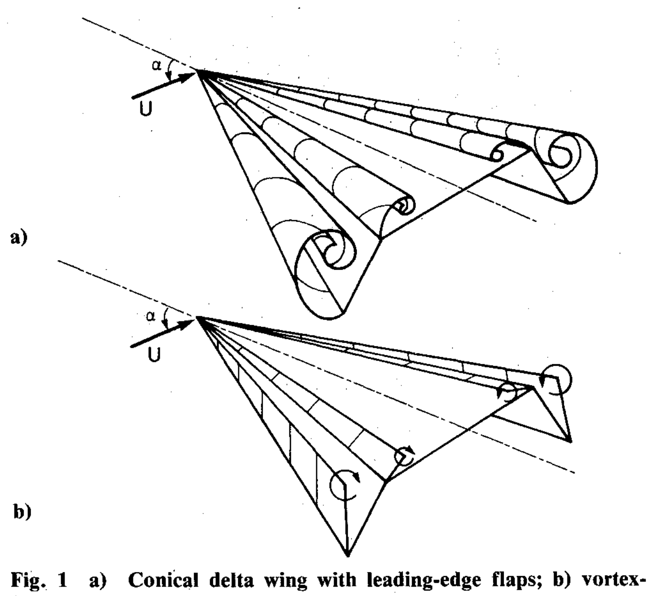

Fig. 1 a) Conical delta wing with leading-edge flaps; b) vortex-feeding-sheet model.

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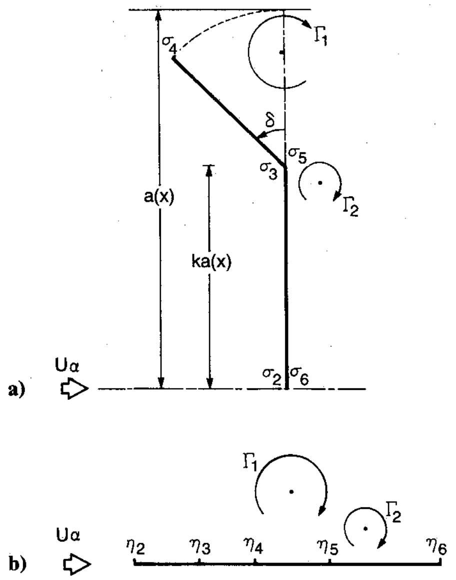

Fig. 2 a) Physical plane; b) transformed plane

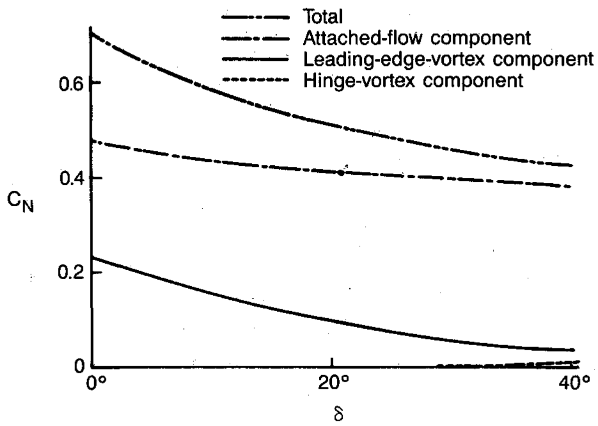

Fig. 3 Normal force coefficient components: $\epsilon = 16$ deg, $k = 0.6$, $\alpha = 10$ deg.

and the complex velocity in the physical plane, $\bar{V} = \mathrm{d}F/\mathrm{d}\sigma$, is

$$\bar{V} = \left[U\alpha + \frac{i\Gamma_1}{2\pi}\left(\frac{1}{\eta-\eta_{v1}} - \frac{1}{\eta-\bar{\eta}_{v1}}\right) + \frac{i\Gamma_2}{2\pi}\left(\frac{1}{\eta-\eta_{v2}} - \frac{1}{\eta-\bar{\eta}_{v2}}\right)\right]\frac{\mathrm{d}\eta}{\mathrm{d}\sigma} \tag{6}$$

The term $\mathrm{d}\eta/\mathrm{d}\sigma$ is singular at the leading edge ($\eta = 0$) and at the flap hinge ($\eta = \eta_5$). To relieve these singularities, two Kutta conditions are invoked. These conditions, together with the equilibrium relations resulting form applying Eq. (1) to the two vortex-feeding-sheet arrangements, give a system of four equations from which the vortex positions η_{v1}, η_{v2} and the vortex strengths Γ_1, Γ_2 can be calculated.

Lift and drag forces on the wing components are obtained by pressure integration. On the main wing,

$$L_w \approx R_w \tag{7a}$$

$$D_w \approx R_w \alpha \tag{7b}$$

Since the flap hinge makes an angle ϵ with the wing centerline, the forces on the flap are

$$L_F \approx R_{Fy}(1 + \epsilon\ \alpha\ \tan\delta) \tag{8a}$$

$$D_F \approx \alpha R_{Fy}\left(1 - \frac{\epsilon}{\alpha}\tan\delta\right) \tag{8b}$$

The total lift-to-drag ratio is

$$\frac{L}{D} = \frac{1}{\alpha}\left[\left(1 + \frac{R_{Fy}}{R_w + R_{Fy}}\frac{\epsilon}{\alpha}\tan\delta\right) + \mathcal{O}(\alpha^2)\right] \tag{9}$$

The terms containing $\tan\delta$ in Eqs. (8) and (9) indicate the effect of the propulsive component of the force acting on the deflected flaps. For $\delta > 0$, L and D decrease while L/D increases. These same trends have been observed by Lamar and Campbell[2] and Hoffler and Rao.[3]

Calling N the resultant force normal to the main wing, the decrement in lift is best understood by considering the projection of N onto the normal to the freestream. N can be easily computed by equating it to the downward momentum flux through the cross-flow plane.

$$N = \rho U\ \mathrm{Im}[\Gamma_1(\eta_{v1} - \bar{\eta}_{v1}) - \Gamma_2(\eta_{v2} - \bar{\eta}_{v2})] - \rho U^2 \alpha\ \mathrm{Im}\int_c \frac{\eta^2}{\sqrt{(\eta-\eta_6)(\eta-\eta_2)}}\left(\frac{\eta-\eta_5}{\eta-\eta_3}\right)\mathrm{d}\eta \tag{10}$$

In terms of N, the global lift is

$$L \approx N + R_{Fy}\ \epsilon\ \alpha\ \tan\delta \tag{11}$$

The second term in Eq. (11) can be shown to be of order α^2. The first term in the normal force is associated with the singularity system, while the second term represents the normal force that would result in the absence of separation.

Figure 3 shows the trends of each normal force component with flap deflection angle. It can be seen that the main reason for lift reduction is the partial suppression of the leading-edge vortex system.

References

[1]Brown, C. E. and Michael, W. H. Jr., "On Slender Delta Wings with Leading-Edge Separation," NACA TN-3430, March 1955.

[2]Lamar, J. E. and Campbell, J. F., "Vortex Flaps—Advanced Control Devices for Supercruise Fighters," *Aerospace America*, Jan. 1984, pp. 95–99.

[3]Hoffler, K. D. and Rao, D. M., "An Investigation of the Tabbed Vortex Flap," *Journal of Aircraft*, Vol. 22, June 1985, pp. 490–497.